to choose from in constructing a course that meets the requirements of the syllabus. The other extreme implies a massive effort to prepare films, television programs, instructional programs for teaching machines, and so on, and to teach the teacher how to use these with wisdom and understanding of the subject. The debate is sufficiently intense and its implications for a philosophy of education sufficiently great that the concluding chapter is devoted to this issue.

In sum, then, we shall concentrate on four themes and one conjecture: the themes of structure, readiness, intuition, and interest, and the conjecture of how best to aid the teacher in the task of instruction.

Bruner, Jerome. (1977 reprint). The Process of Education. Cambridge, MA: Harvard University Press

2

THE IMPORTANCE OF STRUCTURE

The first object of any act of learning, over and beyond the pleasure it may give, is that it should serve us in the future. Learning should not only take us somewhere; it should allow us later to go further more easily. There are two ways in which learning serves the future. One is through its specific applicability to tasks that are highly similar to those we originally learned to perform. Psychologists refer to this phenomenon as specific transfer of training; perhaps it should be called the extension of habits or associations. Its utility appears to be limited in the main to what we usually speak of as skills. Having learned how to hammer nails, we are better able later to learn how to hammer tacks or chip wood. Learning in school undoubtedly creates skills of a kind that transfers to activities encountered later, either in school or after. A second way in which earlier learning renders later performance more efficient is through what is conveniently called nonspecific transfer or, more accurately, the transfer of principles and attitudes. In essence, it consists of learning initially not a skill but a general idea, which can then be used as a basis for recognizing subsequent problems as special cases of the idea originally mastered. This type of transfer is at the heart of the educational process—the continual broadening and deepening of knowledge in terms of basic and general ideas.

The continuity of learning that is produced by the second type of transfer, transfer of principles, is dependent upon mastery of the structure of the subject matter, as structure was described in the preceding chapter. That is to say, in order for a person to be able to recognize the applicability or inapplicability of an idea to a new situation and to broaden his learning thereby, he must have clearly in mind the general nature of the phenomenon with which he is dealing. The more fundamental or basic is the idea he has learned, almost by definition, the greater will be its breadth of applicability to new problems. Indeed, this is almost a tautology, for what is meant by "fundamental" in this sense is precisely that an idea has wide as well as powerful applicability. It is simple enough to proclaim, of course, that school curricula and methods of teaching should be geared to the teaching of fundamental ideas in whatever subject is being taught. But as soon as one makes such a statement a host of problems arise, many of which can be solved only with the aid of considerably more research. We turn to some of these now.

The first and most obvious problem is how to construct curricula that can be taught by ordinary teachers to ordinary students and that at the same time reflect clearly the basic or underlying principles of various fields of inquiry. The problem is twofold: first, how to have the basic subjects rewritten and their teaching materials revamped in such a way that the pervading and powerful ideas and attitudes relating to them are given a central role; second, how to match the levels of these materials to the capacities of students of different abilities at different grades in school.

The experience of the past several years has taught at least one important lesson about the design of a curriculum that is true to the underlying structure of its subject matter. It is that the best minds in any particular discipline must be put to work on the task. The decision as to what should be taught in American history to elementary school children or what should be taught in arithmetic is a decision that can best be reached with the aid of those with a high degree of vision and competence in each of these fields. To decide that the elementary ideas of algebra depend upon the fundamentals of the commutative, distributive, and associative laws, one must be a mathematician in a position to appreciate and understand the fundamentals of mathematics. Whether schoolchildren require an understanding of Frederick Jackson Turner's ideas about the role of the frontier in American history before they can sort out the facts and trends of American history—this again is a decision that requires the help of the scholar who has a deep understanding of the American past. Only by the use of our best minds in devising curricula will we bring the fruits of scholarship and wisdom to the student just beginning his studies.

The question will be raised, "How enlist the aid of our most able scholars and scientists in designing curricula for primary and secondary schools?" The answer has already been given, at least in part. The School Mathematics Study Group, the University of Illinois mathematics projects, the Physical Science Study Committee, and the Biological Sciences Curriculum Study have indeed been enlisting the aid of eminent men in their various fields, doing so by means of summer projects, supplemented in part by year-long leaves of absence for certain key people involved. They have been aided in these projects by outstanding elementary and secondary school teachers and, for special purposes, by professional writers, film makers, designers, and others required in such a complex enterprise.

There is at least one major matter that is left unsettled even by a large-scale revision of curricula in the direction indicated. Mastery of the fundamental ideas of a field involves not only the grasping of general principles, but also the development of an attitude toward learning and inquiry, toward guessing and hunches, toward the possibility of solving problems on one's own. Just as a physicist has certain attitudes about the ultimate orderliness of nature and a conviction that order can be discovered, so a young physics student needs some working version of these attitudes if he is to organize his learning in such a way as to make what he learns usable and meaningful in his thinking. To instill such attitudes by teaching requires something more than the mere presentation of fundamental ideas. Just what it takes to bring off such teaching is something on which a great deal of research is needed, but it would seem that an important ingredient is a sense of excitement about discovery—discovery of regularities of previously unrecognized relations and similarities between ideas, with a resulting sense of self-confidence in one's abilities. Various people who have worked on curricula in science and mathematics have urged that it is possible to present the fundamental structure of a discipline in such a way as to preserve some of the exciting sequences that lead a student to discover for himself.

It is particularly the Committee on School Mathematics and the Arithmetic Project of the University of Illinois that have emphasized the importance of discovery as an aid to teaching. They have been active in devising methods that permit a student to discover for himself the generalization that lies behind a particular mathematical operation, and they contrast this approach with the "method of assertion and proof" in which the generalization is first stated by the teacher and the class asked to proceed through the proof. It has also been pointed out by the Illinois group that the method of discovery would be too time-consuming for presenting all of what a student must cover in mathematics. The proper balance between the two is anything but plain, and research is in progress to elucidate the matter, though more is needed. Is the inductive approach a better technique for teaching principles? Does it have a desirable effect on attitudes?

That the method of discovery need not be limited to such highly formalized subjects as mathematics and physics is illustrated by some experimentation on social studies carried out by the Harvard Cognition Project. A sixth-grade class, having been through a conventional unit on the social and economic geography of the Southeastern states, was introduced to the North Central region by being asked to locate the major cities of the area on a map containing physical features and natural resources, but no place names. The resulting class discussion very rapidly produced a variety of plausible theories concerning the requirements of a city—a water transportation theory that placed Chicago at the junction of the three lakes, a mineral resources theory that placed it near the Mesabi range, a food-supply theory that put a great city on the rich soil of Iowa, and so on. The level of interest as well as the level of conceptual sophistication was far above that of control classes. Most striking, however, was the attitude of children to whom, for the first time, the location of a city appeared as a problem, and one to which an answer could be discovered by taking thought. Not only was there pleasure and excitement in the pursuit of a question, but in the end the discovery was worth making, at least for urban children for whom the phenomenon of the city was something that had before been taken for granted.

How do we tailor fundamental knowledge to the interests and capacities of children? This is a theme we shall return to later, and only a word need be said about it here. It requires a combination of deep understanding and patient honesty to present physical or any other phenomena in a way that is simultaneously exciting, correct, and rewardingly comprehensible. In examining certain teaching materials in physics, for example, we have found much patient honesty in presentation that has come to naught because the authors did not have a deep enough understanding of the subject they were presenting.

A good case in point is to be found in the usual attempt to explain the nature of tides. Ask the majority of high school students to explain tides and they will speak of the gravitational pull of the moon on the surface of the earth and how it pulls the water on the moon's side into a bulge. Ask them now why there is also a bulge of less magnitude on the side of the earth opposite to the moon, and they will almost always be without a satisfactory

answer. Or ask them where the maximum bulge of the incoming tide is with respect to the relative position of the earth and moon, and the answer will usually be that it is at the point on the earth's surface nearest to the moon. If the student knows there is a lag in the tidal crest, he will usually not know why. The failure in both cases comes from an inadequate picture of how gravity acts upon a free-moving elastic body, and a failure to connect the idea of inertia with the idea of gravitational action. In short, the tides are explained without a share of the excitement that can come from understanding Newton's great discovery of universal gravitation and its mode of action. Correct and illuminating explanations are no more difficult and often easier to grasp than ones that are partly correct and therefore too complicated and too restricted. It is the consensus of virtually all the men and women who have been working on curriculum projects that making material interesting is in no way incompatible with presenting it soundly; indeed, a correct general explanation is often the most interesting of all. Inherent in the preceding discussions are at least four general claims that can be made for teaching the fundamental structure of a subject, claims in need of detailed study.

The first is that understanding fundamentals makes a subject more comprehensible. This is true not only in physics and mathematics, where we have principally illustrated the point, but equally in the social studies and literature. Once one has grasped the fundamental idea that a nation must trade in order to live, then such a presumably special phenomenon as the Triangular Trade of the American colonies becomes altogether simpler to understand as something more than commerce in molasses, sugar cane, rum, and slaves in an atmosphere of violation of British trade regulations. The high school student reading *Moby Dick* can only understand more deeply if he can be led to understand that Melville's novel is, among other things, a study of the theme of evil and the plight of those pursuing this "killing whale." And if the student is led further to understand that there are a relatively limited number of human plights about which novels are written, he understands literature the better for it.

The second point relates to human memory. Perhaps the most basic thing that can be said about human memory, after a century of intensive research, is that unless detail is placed into a structured pattern, it is rapidly forgotten. Detailed material is conserved in memory by the use of simplified ways of representing it. These simplified representations have what may be called a "regenerative" character. A good example of this regenerative property of long-term memory can be found in science. A scientist does not try to remember the distances traversed by falling bodies in different gravitational fields over different periods of time. What he carries in memory instead is a formula that permits him with varying degrees of accuracy to regenerate the details on which the more easily remembered formula is based. So he commits to memory the formula $s = \frac{1}{2} gt^2$ and not a handbook of distances, times, and gravitational constants. Similarly, one does not remember exactly what Marlowe, the commentator in *Lord Jim*, said about the chief protagonist's plight, but, rather, simply that he was the dispassionate onlooker, the man who tried to understand without judging what had led Lord Jim into

the straits in which he found himself. We remember a formula, a vivid detail that carries the meaning of an event, an average that stands for a range of events, a caricature or picture that preserves an essence—all of them techniques of condensation and representation. What learning general or fundamental principles does is to ensure that memory loss will not mean total loss, that what remains will permit us to reconstruct the details when needed. A good theory is the vehicle not only for understanding a phenomenon now but also for remembering it tomorrow.

Third, an understanding of fundamental principles and ideas, as noted earlier, appears to be the main road to adequate "transfer of training." To understand something as a specific instance of a more general case—which is what understanding a more fundamental principle or structure means—is to have learned not only a specific thing but also a model for understanding other things like it that one may encounter. If a student could grasp in its most human sense the weariness of Europe at the close of the Thirty Years' War and how it created the conditions for a workable but not ideologically absolute Treaty of Westphalia, he might be better able to think about the ideological struggle of East and West—though the parallel is anything but exact. A carefully wrought understanding should also permit him to recognize the limits of the generalization as well. The idea of "principles" and "concepts" as a basis for transfer is hardly new. It is much in need of more research of a specific kind that would provide detailed knowledge of how best to proceed in the teaching of different subjects in different grades.

The fourth claim for emphasis on structure and prinonly when the student himself tries his hand at writing in different styles. Indeed, it is the underlying premise of laboratory exercises that doing something helps one understand it. There is a certain wisdom in the quip made by a psychologist at Woods Hole: "How do I know what I think until I feel what I do?" In any case, the distinction is not a very helpful one. What is more to the point is to ask what methods of exercise in any given field are most likely to give the student a sense of intelligent mastery over the material. What are the most fruitful computational exercises that one can use in various branches of mathematics? Does the effort to write in the style of Henry James give one an especially good insight into that author's style? Perhaps a good start toward understanding such matters would be to study the methods used by successful teachers. It would be surprising if the information compiled failed to suggest a host of worthwhile laboratory studies on techniques of teaching—or, indeed, on techniques of imparting complex information generally.

A word is needed, finally, on examinations. It is obvious that an examination can be bad in the sense of emphasizing trivial aspects of a subject. Such examinations can encourage teaching in a disconnected fashion and learning by rote. What is often overlooked, however, is that examinations can also be allies in the battle to improve curricula and teaching. Whether an examination is of the "objective" type involving multiple choices or of the essay type, it can be devised so as to emphasize an understanding of the broad principles of a subject. Indeed, even when one examines on detailed knowledge, it can be done in such a way as to require an understanding by the student of the connectedness between specific facts. There is a concerted effort now under way among national testing organizations like the Educational Testing Service to construct examinations that will emphasize an understanding of fundamental principles. Such efforts can be of great help. Additional help might be given to local school systems by making available to them manuals that describe the variety of ways in which examinations can be constructed. The searching examination is not easy to make, and a thoughtful manual on the subject would be welcome.

To recapitulate, the main theme of this chapter has been that the curriculum of a subject should be determined by the most fundamental understanding that can be achieved of the underlying principles that give structure to that subject. Teaching specific topics or skills without making clear their context in the broader fundamental structure of a field of knowledge is uneconomical in several deep senses. In the first place, such teaching makes it exceedingly difficult for the student to generalize from what he has learned to what he will encounter later. In the second place, learning that has fallen short of a grasp of general principles has little reward in terms of intellectual excitement. The best way to create interest in a subject is to render it worth knowing, which means to make the knowledge gained usable in one's thinking beyond the situation in which the learning has occurred. Third, knowledge one has acquired without sufficient structure to tie it together is knowledge that is likely to be forgotten. An unconnected set of facts has a pitiably short half-life in memory. Organizing facts in terms of principles and ideas from which they may be inferred is the only known way of reducing the quick rate of loss of human memory.

Designing curricula in a way that reflects the basic structure of a field of knowledge requires the most fundamental understanding of that field. It is a task that cannot be carried out without the active participation of the ablest scholars and scientists. The experience of the past several years has shown that such scholars and scientists, working in conjunction with experienced teachers and students of child development, can prepare curricula of the sort we have been considering. Much more effort in the actual preparation of curriculum materials, in teacher training, and in supporting research will be necessary if improvements in our educational practices are to be of an order that will meet the challenges of the scientific and social revolution through which we are now living.

There are many problems of how to teach general principles in a way that will be both effective and interesting, and several of the key issues have been passed in review. What is abundantly clear is that much work remains to be done by way of examining currently effective practices, fashioning curricula that may be tried out on an experimental basis, and carrying out the kinds of research that can give support and guidance to the general effort at improving teaching.

How may the kind of curriculum we have been discussing be brought within the intellectual reach of children of different ages? To this problem we turn next.

3

READINESS FOR LEARNING

We begin with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development. It is a bold hypothesis and an essential one in thinking about the nature of a curriculum. No evidence exists to contradict it; considerable evidence is being amassed that supports it.

To make clear what is implied, let us examine three general ideas. The first has to do with the process of intellectual development in children, the second with the act of learning, and the third with the notion of the "spiral curriculum" introduced earlier.

Intellectual development. Research on the intellectual development of the child highlights the fact that at each stage of development the child has a characteristic way of viewing the world and explaining it to himself. The task of teaching a subject to a child at any particular age is one of representing the structure of that subject in terms of the child's way of viewing things. The task can be thought of as one of translation. The general hypothesis that has just been stated is premised on the considered judgment that any idea can be represented honestly and usefully in the thought forms of children of school age, and that these first representations can later be made more powerful and precise the more easily by virtue of this early learning. To illustrate and support this view, we present here a somewhat detailed picture of the course of intellectual development, along with some suggestions about teaching at different stages of it.

The work of Piaget and others suggests that, roughly speaking, one may distinguish three stages in the intellectual development of the child. The first stage need not concern us in detail, for it is characteristic principally of the pre-school child. In this stage, which ends (at least for Swiss school children) around the fifth or sixth year, the child's mental work consists principally in establishing relationships between experience and action; his concern is with manipulating the world through action. This stage corresponds roughly to the period from the first development of language to the point at which the child learns to manipulate symbols. In this so-called preoperational stage, the principal symbolic achievement is that the child learns how to represent the external world through symbols established by simple generalization; things are represented as equivalent in terms of sharing some common property. But the child's symbolic world does not make a clear separation between internal motives and feelings on the one hand and external reality on the other. The sun moves because God pushes it, and the stars, like himself, have to go to bed. The child is little able to separate his own goals from the means for achieving them, and when he has to make corrections in his activity after unsuccessful attempts at manipulating reality, he does so by what are called intuitive regulations rather than by symbolic operations, the former being of a crude trial-and-error nature rather than the result of taking thought.

What is principally lacking at this stage of develop-

ment is what the Geneva school has called the concept of reversibility. When the shape of an object is changed, as when one changes the shape of a ball of plasticene, the preoperational child cannot grasp the idea that it can be brought back readily to its original state. Because of this fundamental lack the child cannot understand certain fundamental ideas that lie at the basis of mathematics and physics—the mathematical idea that one conserves quantity even when one partitions a set of things into subgroups, or the physical idea that one conserves mass and weight even though one transforms the shape of an object. It goes without saying that teachers are severely limited in transmitting concepts to a child at this stage, even in a highly intuitive manner.

The second stage of development—and now the child is in school—is called the stage of concrete operations. This stage is operational in contrast to the preceding stage, which is merely active. An operation is a type of action: it can be carried out rather directly by the manipulation of objects, or internally, as when one manipulates the symbols that represent things and relations in one's mind. Roughly, an operation is a means of getting data about the real world into the mind and there transforming them so that they can be organized and used selectively in the solution of problems. Assume a child is presented with a pinball machine which bounces a ball off a wall at an angle. Let us find out what he appreciates about the relation between the angle of incidence and the angle of reflection. The young child sees no problem: for him, the ball travels in an arc, touching the wall on the [illegible] somewhat older child, say age ten, sees the two angles as roughly related

—as one changes so does the other. The still older child begins to grasp that there is a fixed relation between the two, and usually says it is a right angle. Finally, the thirteen- or fourteen-year-old, often by pointing the ejector directly at the wall and seeing the ball come back at the ejector, gets the idea that the two angles are equal. Each way of looking at the phenomenon represents the result of an operation in this sense, and the child's thinking is constrained by his way of pulling his observations together.

An operation differs from simple action or goal-directed behavior in that it is internalized and reversible. "Internalized" means that the child does not have to go about his problem-solving any longer by overt trial and error, but can actually carry out trial and error in his head. Reversibility is present because operations are seen as characterized where appropriate by what is called "complete compensation"; that is to say, an operation can be compensated for by an inverse operation. If marbles, for example, are divided into subgroups, the child can grasp intuitively that the original collection of marbles can be restored by being added back together again. The child tips a balance scale too far with a weight and then searches systematically for a lighter weight or for something with which to get the scale rebalanced. He may carry reversibility too far by assuming that a piece of paper, once burned, can also be restored.

With the advent of concrete operations, the child develops an internalized structure with which to operate. In the example of the balance scale, the structure is a serial order of weights that the child has in his mind.

Such internal structures are of the essence. They are the internalized symbolic systems by which the child represents the world, as in the example of the pinball machine and the angles of incidence and reflection. It is into the language of these internal structures that one must translate ideas if the child is to grasp them.

But concrete operations, though they are guided by the logic of classes and the logic of relations, are means for structuring only immediately present reality. The child is able to give structure to the things he encounters, but he is not yet readily able to deal with possibilities not directly before him or not already experienced. This is not to say that children operating concretely are not able to anticipate things that are not present. Rather, it is that they do not command the operations for conjuring up systematically the full range of alternative possibilities that could exist at any given time. They cannot go systematically beyond the information given them to a description of what else might occur. Somewhere between ten and fourteen years of age the child passes into a third stage, which is called the stage of "formal operations" by the Geneva school.

Now the child's intellectual activity seems to be based upon an ability to operate on hypothetical propositions rather than being constrained to what he has experienced or what is before him. The child can now think of possible variables and even deduce potential relationships that can later be verified by experiment or observation. Intellectual operations now appear to be predicated upon the same kinds of logical operations that are the stock in trade of the logician, the scientist, or the abstract thinker. It is at this point that the child is able to give formal or axiomatic expression to the concrete ideas that before guided his problem-solving but could not be described or formally understood.

Earlier, while the child is in the stage of concrete operations, he is capable of grasping intuitively and concretely a great many of the basic ideas of mathematics, the sciences, the humanities, and the social sciences. But he can do so only in terms of concrete operations. It can be demonstrated that fifth-grade children can play mathematical games with rules modeled on highly advanced mathematics; indeed, they can arrive at these rules inductively and learn how to work with them. They will flounder, however, if one attempts to force upon them a formal mathematical description of what they have been doing, though they are perfectly capable of guiding their behavior by these rules. At the Woods Hole Conference we were privileged to see a demonstration of teaching in which fifth-grade children very rapidly grasped central ideas from the theory of functions, although had the teacher attempted to explain to them what the theory of functions was, he would have drawn a blank. Later, at the appropriate stage of development and given a certain amount of practice in concrete operations, the time would be ripe for introducing them to the necessary formalism.

What is most important for teaching basic concepts is that the child be helped to pass progressively from concrete thinking to the utilization of more conceptually adequate modes of thought. But it is futile to attempt this by presenting formal explanations based on a logic that is distant from the child's manner of thinking and sterile in its implications for him. Much teaching in

mathematics is of this sort. The child learns not to understand mathematical order but rather to apply certain devices or recipes without understanding their significance and connectedness. They are not translated into his way of thinking. Given this inappropriate start, he is easily led to believe that the important thing is for him to be "accurate"—though accuracy has less to do with mathematics than with computation. Perhaps the most striking example of this type of thing is to be found in the manner in which the high school student meets Euclidian geometry for the first time, as a set of axioms and theorems, without having had some experience with simple geometric configurations and the intuitive means whereby one deals with them. If the child were earlier given the concepts and strategies in the form of intuitive geometry at a level that he could easily follow, he might be far better able to grasp deeply the meaning of the theorems and axioms to which he is exposed later.

But the intellectual development of the child is no clockwork sequence of events; it also responds to influences from the environment, notably the school environment. Thus instruction in scientific ideas, even at the elementary level, need not follow slavishly the natural course of cognitive development in the child. It can also lead intellectual development by providing challenging but usable opportunities for the child to forge ahead in his development. Experience has shown that it is worth the effort to provide the growing child with problems that tempt him into next stages of development. As David Page, one of the most experienced teachers of elementary mathematics, has commented: "In teaching from kindergarten to graduate school, I have been amazed at the intellectual similarity of human beings at all ages, although children are perhaps more spontaneous, creative, and energetic than adults. As far as I am concerned young children learn almost anything faster than adults do if it can be given to them in terms they understand. Giving the material to them in terms they understand, interestingly enough, turns out to involve knowing the mathematics oneself, and the better one knows it, the better it can be taught. It is appropriate that we warn ourselves to be careful of assigning an absolute level of difficulty to any particular topic. When I tell mathematicians that fourth-grade students can go a long way into 'set theory' a few of them reply: 'Of course.' Most of them are startled. The latter ones are completely wrong in assuming that 'set theory' is intrinsically difficult. Of course it may be that nothing is intrinsically difficult. We just have to wait until the proper point of view and corresponding language for presenting it are revealed. Given particular subject matter or a particular concept, it is easy to ask trivial questions or to lead the child to ask trivial questions. It is also easy to ask impossibly difficult questions. The trick is to find the medium questions that can be answered and that take you somewhere. This is the big job of teachers and textbooks." One leads the child by the well-wrought "medium questions" to move more rapidly through the stages of intellectual development, to a deeper understanding of mathematical, physical, and historical principles. We must know far more about the ways in which this can be done.

Professor Inhelder of Geneva was asked to suggest ways in which the child could be moved along faster

through the various stages of intellectual development in mathematics and physics. What follows is part of a memorandum she prepared for the Conference.

"The most elementary forms of reasoning—whether logical, arithmetical, geometrical, or physical—rest on the principle of the invariance of quantities: that the whole remains, whatever may be the arrangement of its parts, the change of its form, or its displacement in space or time. The principle of invariance is no a priori datum of the mind, nor is it the product of purely empirical observation. The child discovers invariance in a manner comparable to scientific discoveries generally. Grasping the idea of invariance is beset with difficulties for the child, often unsuspected by teachers. To the young child, numerical wholes, spatial dimensions, and physical quantities do not seem to remain constant but to dilate or contract as they are operated upon. The total number of beads in a box remains the same whether subdivided into two, three, or ten piles. It is this that is so hard for the child to understand. The young child perceives changes as operating in one direction without being able to grasp the idea that certain fundamental features of things remain constant over change, or that if they change the change is reversible.

"A few examples among many used in studying the child's concept of invariance will illustrate the kinds of materials one could use to help him to learn the concept more easily. The child transfers beads of a known quantity or liquids of a known volume from one receptacle to another, one receptacle being tall and narrow, the other flat and wide. The young child believes there is more in the tall receptacle than the flat one. Now the child can be confronted concretely with the nature of one-to-one correspondence between two versions of the same quantity. For there is an easy technique of checking: the beads can be counted or the liquid measured in some standard way. The same operations work for the conservation of spatial quantity if one uses a set of sticks for length or a set of tiles for surface, or by having the child transform the shape of volumes made up of the same number of blocks. In physics dissolving sugar or transforming the shapes of balls of plasticene while conserving volume provides comparable instruction. If teaching fails to bring the child properly from his perceptual, primitive notions to a proper intuition of the idea of invariance, the result is that he will count without having acquired the idea of the invariance of numerical quantities. Or he will use geometrical measures while remaining ignorant of the operation of transitivity—that if A includes B, and B includes C, then A also includes C. In physics he will apply calculations to imperfectly understood physical notions such as weight, volume, speed, and time. A teaching method that takes into account the natural thought processes will allow the child to discover such principles of invariance by giving him an opportunity to progress beyond his own primitive mode of thinking through confrontation by concrete data—as when he notes that liquid that looks greater in volume in a tall, thin receptacle is in fact the same as that quantity in a flat, low vessel. Concrete activity that becomes increasingly formal is what leads the child to the kind of mental mobility that approaches the naturally reversible operations of mathematics and logic. The child gradually comes to sense that any change may be mentally cancelled out by the reverse operation—addition by subtraction—or that a change may be counterbalanced by a reciprocal change.

"A child often focuses on only one aspect of a phenomenon at a time, and this interferes with his understanding. We can set up little teaching experiments in such a way that he is forced to pay attention to other aspects. Thus, children up to about age seven estimate the speed of two automobiles by assuming that the one that gets there first is the faster, or that if one passes the other it is faster. To overcome such errors, one can, by using toy automobiles, show that two objects starting at different distances from a finish line cannot be judged by which one arrives first, or show that one car can pass another by circling it and still not finish first. These are simple exercises, but they speed the child toward attending to several features of a situation at once.

"In view of all this it seems highly arbitrary and very likely incorrect to delay the teaching, for example, of Euclidian or metric geometry until the end of the primary grades, particularly when projective geometry has not been given earlier. So too with the teaching of physics, which has much in it that can be profitably taught at an inductive or intuitive level much earlier. Basic notions in these fields are perfectly accessible to children of seven to ten years of age, *provided that they are divorced from their mathematical expression and studied through materials that the child can handle himself*.

"Another matter relates particularly to the ordering of a mathematics curriculum. Often the sequence of psychological development follows more closely the axiomatic order of a subject matter than it does the historical order of development of concepts within the field. One observes, for instance, that certain topological notions, such as connection, separation, being interior to, and so forth, precede the formation of Euclidian and projective notions in geometry, though the former ideas are newer in their formalism in the history of mathematics than the latter. If any special justification were needed for teaching the structure of a subject in its proper logical or axiomatic order rather than its order of historical development, this should provide it. This is not to say that there may not be situations where the historical order is important from the point of view of its cultural or pedagogical relevance.

"As for teaching geometrical notions of perspective and projection, again there is much that can be done by the use of experiments and demonstrations that rest on the child's operational capacity to analyze concrete experience. We have watched children work with an apparatus in which rings of different diameter are placed at different positions between a candle and a screen with a fixed distance between them so that the rings cast shadows of varying sizes on the screen. The child learns how the cast shadow changes size as a function of the distance of the ring from the light source. By bringing to the child such concrete experience of light in revealing situations, we teach him maneuvers that in the end permit him to understand the general ideas underlying projective geometry.

"These examples lead us to think that it is possible to draw up methods of teaching the basic ideas in science and mathematics to children considerably younger than the traditional age. It is at this earlier age that systematic

instruction can lay a groundwork in the fundamentals that can be used later and with great profit at the secondary level.

"The teaching of probabilistic reasoning, so very common and important a feature of modern science, is hardly developed in our educational system before college. The omission is probably due to the fact that school syllabi in nearly all countries follow scientific progress with a near-disastrous time lag. But it may also be due to the widespread belief that the understanding of random phenomena depends on the learner's grasp of the meaning of the rarity or commonness of events. And admittedly, such ideas are hard to get across to the young. Our research indicates that the understanding of random phenomena requires, rather, the use of certain concrete logical operations well within the grasp of the young child—provided these operations are free of awkward mathematical expression. Principal among these logical operations are disjunction ('either A *or* B is true') and combination. Games in which lots are drawn, games of roulette, and games involving a gaussian distribution of outcomes are all ideal for giving the child a basic grasp of the logical operation needed for thinking about probability. In such games, children first discover an entirely qualitative notion of chance defined as an uncertain event, contrasted with deductive certainty. The notion of probability as a fraction of certainty is discovered only later. Each of these discoveries can be made before the child ever learns the techniques of the calculus of probabilities or the formal expressions that normally go with probability theory. Interest in problems of a probabilistic nature could easily be awakened and developed before the introduction of any statistical processes or computation. Statistical manipulation and computation are only tools to be used *after* intuitive understanding has been established. If the array of computational paraphernalia is introduced first, then more likely than not it will inhibit or kill the development of probabilistic reasoning.

"One wonders in the light of all this whether it might not be interesting to devote the first two years of school to a series of exercises in manipulating, classifying, and ordering objects in ways that highlight basic operations of logical addition, multiplication, inclusion, serial ordering, and the like. For surely these logical operations are the basis of more specific operations and concepts of all mathematics and science. It may indeed be the case that such an early science and mathematics 'pre-curriculum' might go a long way toward building up in the child the kind of intuitive and more inductive understanding that could be given embodiment later in formal courses in mathematics and science. The effect of such an approach would be, we think, to put more continuity into science and mathematics and also to give the child a much better and firmer comprehension of the concepts which, unless he has this early foundation, he will mouth later without being able to use them in any effective way."

A comparable approach can surely be taken to the teaching of social studies and literature. There has been little research done on the kinds of concepts that a child brings to these subjects, although there is a wealth of observation and anecdote. Can one teach the structure of literary forms by presenting the child with the first

part of a story and then having him complete it in the form of a comedy, a tragedy, or a farce—without ever using such words? When, for example, does the idea of "historical trend" develop, and what are its precursors in the child? How does one make a child aware of literary style? Perhaps the child can discover the idea of style through the presentation of the same content written in drastically different styles, in the manner of Beerbohm's *Christmas Garland*. Again, there is no reason to believe that any subject cannot be taught to any child at virtually any age in some form.

Here one is immediately faced with the question of the economy of teaching. One can argue that it might be better to wait until the child is thirteen or fourteen before beginning geometry so that the projective and intuitive first steps can immediately be followed up by a full formal presentation of the subject. Is it worth while to train the young inductively so that they may discover the basic order of knowledge before they can appreciate its formalism? In Professor Inhelder's memorandum, it was suggested that the first two grades might be given over to training the child in the basic logical operations that underlie instruction in mathematics and science. There is evidence to indicate that such rigorous and relevant early training has the effect of making later learning easier. Indeed the experiments on "learning set" seem to indicate just that—that one not only learns specifics but in so doing learns how to learn. So important is training per se that monkeys who have been given extensive training in problem solving suffer considerably less loss and recover more quickly after induced brain damage than animals who had not been previously thus educated. But the danger of such early training may be that it has the effect of training out original but deviant ideas. There is no evidence available on the subject, and much is needed.

The act of learning. Learning a subject seems to involve three almost simultaneous processes. First there is *acquisition* of new information—often information that runs counter to or is a replacement for what the person has previously known implicitly or explicitly. At the very least it is a refinement of previous knowledge. Thus one teaches a student Newton's laws of motion, which violate the testimony of the senses. Or in teaching a student about wave mechanics, one violates the student's belief in mechanical impact as the sole source of real energy transfer. Or one bucks the language and its built-in way of thinking in terms of "wasting energy" by introducing the student to the conservation theorem in physics which asserts that no energy is lost. More often the situation is less drastic, as when one teaches the details of the circulatory system to a student who already knows vaguely or intuitively that blood circulates.

A second aspect of learning may be called *transformation*—the process of manipulating knowledge to make it fit new tasks. We learn to "unmask" or analyze information, to order it in a way that permits extrapolation or interpolation or conversion into another form. Transformation comprises the ways we deal with information in order to go beyond it.

A third aspect of learning is *evaluation:* checking whether the way we have manipulated information is adequate to the task. Is the generalization fitting, have we extrapolated appropriately, are we operating proper-

ly? Often a teacher is crucial in helping with evaluation, but much of it takes place by judgments of plausibility without our actually being able to check rigorously whether we are correct in our efforts.

In the learning of any subject matter, there is usually a series of episodes, each episode involving the three processes. Photosynthesis might reasonably comprise material for a learning episode in biology, fitted into a more comprehensive learning experience such as learning about the conversion of energy generally. At its best a learning episode reflects what has gone before it and permits one to generalize beyond it.

A learning episode can be brief or long, contain many ideas or a few. How sustained an episode a learner is willing to undergo depends upon what the person expects to get from his efforts, in the sense of such external things as grades but also in the sense of a gain in understanding.

We usually tailor material to the capacities and needs of students by manipulating learning episodes in several ways: by shortening or lengthening the episode, by piling on extrinsic rewards in the form of praise and gold stars, or by dramatizing the shock of recognition of what the material means when fully understood. The unit in a curriculum is meant to be a recognition of the importance of learning episodes, though many units drag on with no climax in understanding. There is a surprising lack of research on how one most wisely devises adequate learning episodes for children at different ages and in different subject matters. There are many questions that need answers based on careful research, and to some of these we turn now.

There is, to begin with, the question of the balance between extrinsic rewards and intrinsic ones. There has been much written on the role of reward and punishment in learning, but very little indeed on the role of interest and curiosity and the lure of discovery. If it is our intention as teachers to inure the child to longer and longer episodes of learning, it may well be that intrinsic rewards in the form of quickened awareness and understanding will have to be emphasized far more in the detailed design of curricula. One of the least discussed ways of carrying a student through a hard unit of material is to challenge him with a chance to exercise his full powers, so that he may discover the pleasure of full and effective functioning. Good teachers know the power of this lure. Students should know what it feels like to be completely absorbed in a problem. They seldom experience this feeling in school. Given enough absorption in class, some students may be able to carry over the feeling to work done on their own.

There is a range of problems that have to do with how much emphasis should be placed on acquisition, transformation, and evaluation in a learning episode—getting facts, manipulating them, and checking one's ideas. Is it the case, for example, that it is best to give the young child a minimum set of facts first and then encourage him to draw the fullest set of implications possible from this knowledge? In short, should an episode for a young child contain little new information but emphasize what can be done to go beyond that bit on one's own? One teacher of social studies has had great success with fourth-graders through this approach: he begins, for example, with the fact that civilizations

have most often begun in fertile river valleys—the only "fact." The students are encouraged in class discussion to figure out why this is the case and why it would be less likely for civilizations to start in mountainous country. The effect of this approach, essentially the technique of discovery, is that the child generates information on his own, which he can then check or evaluate against the sources, getting more new information in the process. This obviously is one kind of learning episode, and doubtless it has limited applicability. What other kinds are there, and are some more appropriate to certain topics and ages than others? It is not the case that "to learn is to learn is to learn," yet in the research literature there appears to be little recognition of differences in learning episodes.

With respect to the optimum length of a learning episode, there are a few commonsense things one can say about it, and these are perhaps interesting enough to suggest fruitful research possibilities. It seems fairly obvious, for example, that the longer and more packed the episode, the greater the pay-off must be in terms of increased power and understanding if the person is to be encouraged to move to a next episode with zest. Where grades are used as a substitute for the reward of understanding, it may well be that learning will cease as soon as grades are no longer given—at graduation.

It also seems reasonable that the more one has a sense of the structure of a subject, the more densely packed and longer a learning episode one can get through without fatigue. Indeed, the amount of new information in any learning episode is really the amount that we cannot quite fit into place at once. And there is a severe limit, as we have already noted, on how much of such unassimilated information we can keep in mind. The estimate is that adults can handle about seven independent items of information at a time. No norms are available for children—a deplorable lack.

There are many details one can discuss concerning the shaping of learning episodes for children, but the problems that have been mentioned will suffice to give their flavor. Inasmuch as the topic is central to an understanding of how one arranges a curriculum, it seems obvious that here is an area of research that is of the first importance.

The "spiral curriculum." If one respects the ways of thought of the growing child, if one is courteous enough to translate material into his logical forms and challenging enough to tempt him to advance, then it is possible to introduce him at an early age to the ideas and styles that in later life make an educated man. We might ask, as a criterion for any subject taught in primary school, whether, when fully developed, it is worth an adult's knowing, and whether having known it as a child makes a person a better adult. If the answer to both questions is negative or ambiguous, then the material is cluttering the curriculum.

If the hypothesis with which this section was introduced is true—that any subject can be taught to any child in some honest form—then it should follow that a curriculum ought to be built around the great issues, principles, and values that a society deems worthy of the continual concern of its members. Consider two examples—the teaching of literature and of science. If it is granted, for example, that it is desirable to give

children an awareness of the meaning of human tragedy and a sense of compassion for it, is it not possible at the earliest appropriate age to teach the literature of tragedy in a manner that illumirates but does not threaten? There are many possible ways to begin: through a retelling of the great myths, through the use of children's classics, through presentation of and commentary on selected films that have proved themselves. Precisely what kinds of materials should be used at what age with what effect is a subject for research—research of several kinds. We may ask first about the child's conception of the tragic, and here one might proceed in much the same way that Piaget and his colleagues have proceeded in studying the child's conception of physical causality, of morality, of number, and the rest. It is only when we are equipped with such knowledge that we will be in a position to know how the child will translate whatever we present to him into his own subjective terms. Nor need we wait for all the research findings to be in before proceeding, for a skillful teacher can also experiment by attempting to teach what seems to be intuitively right for children of different ages, correcting as he goes. In time, one goes beyond to more complex versions of the same kind of literature or simply revisits some of the same books used earlier. What matters is that later teaching build upon earlier reactions to literature, that it seek to create an ever more explicit and mature understanding of the literature of tragedy. Any of the great literary forms can be handled in the same way, or any of the great themes—be it the form of comedy or the theme of identity, personal loyalty, or what not.

So too in science. If the understanding of number, measure, and probability is judged crucial in the pursuit of science, then instruction in these subjects should begin as intellectually honestly and as early as possible in a manner consistent with the child's forms of thought. Let the topics be developed and redeveloped in later grades. Thus, if most children are to take a tenth-grade unit in biology, need they approach the subject cold? Is it not possible, with a minimum of formal laboratory work if necessary, to introduce them to some of the major biological ideas earlier, in a spirit perhaps less exact and more intuitive?

Many curricula are originally planned with a guiding idea much like the one set forth here. But as curricula are actually executed, as they grow and change, they often lose their original form and suffer a relapse into a certain shapelessness. It is not amiss to urge that actual curricula be reexamined with an eye to the issues of continuity and development referred to in the preceding pages. One cannot predict the exact forms that revision might take; indeed, it is plain that there is now available too little research to provide adequate answers. One can only propose that appropriate research be undertaken with the greatest vigor and as soon as possible.